An Approximate Statistical Model of the Fairness of the Earliest Deadline First Scheduler

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***Abstract*— This paper is an extension of an earlier paper the authors published, which modelled the probability density function of the queueing delay of Earliest Deadline First traffic. In this paper, we introduce a new definition and derive an expression for the stochastic fairness of real-time traffic classes in an Earliest Deadline First queueing system. The approximation favourably compares with corresponding simulation results. The model has shows that all traffic classes are treated in a similar fashion, although traffic class 2 experiences slightly better and class 0 slightly worse treatment than class 1. Furthermore, the results show that the fairness of EDF can not be bounded deterministically.**

***Index Terms*— Earliest Deadline First (EDF), stochastic fairness, probability density function (pdf).**

I. Introduction

In order to engineer a future network, analytical models are required that predict the behaviour of the various networking components. An exact analysis, as offered by conventional queueing theory, cannot be used to model modern networks carrying heterogeneous traffic, as these networks are generally too complicated. Approximations such as large deviation principles and the effective bandwidth theory offer a simple yet accurate way to mathematically model the network metrics of interest. In this paper we build on the results of [1], where we found an analytical expression for the probability density function of the queueing time that packets in an Earliest Deadline First (EDF) queue experience. We derive a mathematical expression that is able to predict the stochastic fairness distribution of traffic classes in an EDF queue.

Section II introduces the analytical model. A brief overview of the EDF multiplexor is given, followed by a new fairness definition for real-time traffic in Section III.

In Section IV the fairness distribution is derived from the queueing delay probability density function that was found in [1]. Section V-A gives a brief overview of the system parameters used to generate both the analytical and simulation results. Finally, in Section V-B the analytical and simulation results are discussed.

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II. System model

Imagine that the various sources in a network produce data packets of J different classes. Examples of such data types are voice, stored video, video-conference, e-mail, ftp, and http packets. Each of these has different delay, jitter (variation in delay), throughput, bit-error-rate and packet-loss-rate requirements. Only when a network is able to meet all the requirements of all the data classes, can it claim to provide quality-of-service (QoS).

The Earliest Deadline First (EDF) scheduler [2] is one of the simplest multiplexing algorithms, especially well suited for real-time traffic, as it is able to adhere to the delay deadlines that these traffic classes require. Voice and video are examples of real-time traffic, as opposed to best-effort traffic, which includes e-mail, ftp and http. As a packet of data class j is created, it is assigned a corresponding delay deadline d_j, which represents the amount of time that packets of data type j can afford to queue at each switching node they pass through. When packet p_j of data type j arrives at a node at time T_j, it will be time-stamped with a queueing time deadline of $T_j + d_j$, by when the packet will have to be served. At scheduling interval τ (when the previously chosen packet has completed transmission), the EDF scheduler picks the packet from the queue, which has the closest deadline: $\min_j(T_j + d_j - \tau)$.

Adopting the system model of [3], imagine that K sources produce J different data types, with corresponding upper delay bounds of $d_1, d_2, \ldots, d_J$, where $d_1 \leq d_2 \leq \ldots \leq d_J$. Let k_j be the number of sources that produce class j data. C is the link rate at which a chosen packet will be served, while L is the packet size, which we assume to be constant. The amount of work of class j that source i produces in the time interval $[0, t]$ is represented by $A_{ji}[0, t]$.

III. Stochastic fairness definition

One of the first researchers to define fairness mathematically was Golestani. In [4], he defines a perfectly fair system to have the quality

$$w_k(t_1, t_2) = w_j(t_1, t_2), \;\; k, j \in B(t_1, t_2). \tag{1}$$

Here $B(t_1, t_2)$ is the set of sessions that are backlogged during the interval (t_1, t_2), while

$$w_j(t_1, t_2) = \frac{W_k(t_1, t_2)}{r_k} \qquad k \in K, \tag{2}$$

where K is the set of sessions set up on a link. $W_k(t_1, t_2)$ are the number of bits of session k transmitted during $(0, t)$, while r_k represents the service share allocated to session k.

Effectively, what this means is that in a perfectly fair server, the amount of service a session receives normalised by its share of the available bandwidth, should be the same for all backlogged sessions. In reality, no packetised implementation of a fluid system can achieve this goal. Stiliadis in [5] and [6] therefore replaced his equivalent version of the perfect fairness definition

$$\max \left| \frac{W_a^S(\tau, t)}{r_a} - \frac{W_b^S(\tau, t)}{r_b} \right| = 0 \qquad \text{for all } a \text{ and } b, \tag{3}$$

with a more realistic version

$$\max \left| \frac{W_a^S(\tau, t)}{r_a} - \frac{W_b^S(\tau, t)}{r_b} \right| \leq F^S \qquad \text{for all } a \text{ and } b. \tag{4}$$

The bound F^S is an upper bound and indicates how fair a scheduler is.

Stiliadis' definition is based on the assumption that the various traffic classes have been promised a throughput guarantee. This is a good system for best-effort traffic. For real-time traffic, the biggest concern is the delay that packets experience while travelling through the network. A different fairness definition is needed which takes delay deadlines into consideration, instead of throughput guarantees. Furthermore, the definition proposed by Stiliadis is deterministic — the fairness value F^S may never be exceeded. The problem with a definition of this kind is that extremely large fairness values might be possible, but with very low probability. This means that F^S might be situated very far away from the mean fairness value and therefore does not captured the common behaviour of the scheduler. In some cases, the fairness might not be bounded at all, which means that F^S cannot be found.

A stochastic fairness bound, on the other hand, guarantees that the fairness value of class a will not exceed the bound F_a with a probability more than δ [7]. We therefore propose the following expression as a suitable definition for the fairness of real-time class-a traffic:

$$\text{Prob}\left\{ \left| \frac{D_a}{d_a} - \frac{D_b}{d_b} \right| > F_a(\delta) \right\} \leq \delta, \qquad \text{for all } b, \tag{5}$$

where D_a is the delay that a packet of class a with delay deadline d_a experienced, while δ is the maximum probability that the fairness limit F may be exceeded by.

In practice the stochastic fairness $F_a(\delta)$ of *Type* a traffic can be found by rewriting (5) in integral form

$$\int_{F_a(\delta)}^{\infty} f_{|\frac{D_a}{d_a} - \frac{D_b}{d_b}|}(t)\, dt = \delta. \tag{6}$$

Once this expression is found, we would like to plot the fairness $F_a(\delta)$ for varying values of δ.

IV. Derivation of the stochastic fairness expression

In this paper we extend the work that we presented in [1], where we showed that the probability density function of the queueing time that a packet p_i belonging to class i traffic can expect in an EDF queue is suitably approximated by

$$\begin{aligned} f_i(t) = &\left(\delta C - (e^\delta - 1) \sum_{j=1}^{h_i(t)} k_j \lambda_j \right) \\ &\cdot \exp\Bigg[-\delta C t + (e^\delta - 1) \sum_{j=1}^{h_i(t)} k_j \lambda_j t \\ &\qquad + (e^\delta - 1) \sum_{j=h_i(t)+1}^{J} k_j \lambda_j (d_i - d_j) \Bigg]. \end{aligned} \tag{7}$$

where $h_i(t) \leq i$ represents the traffic classes, which might have packets arriving that should be served before packet p_i, but which only arrive once packet p_i has already been transmitted at time t. This can be summarised by

$$\begin{cases} d_i - d_j \geq t, & j \in [1, h_i(t)] \\ d_i - d_j < t, & j \in [h_i(t) + 1, J]. \end{cases} \tag{8}$$

In order to find an expression for the stochastic fairness of real-time traffic as given by 6, we need to find the probability density function (pdf) distribution

$$f_{|\frac{D_a}{d_a} - \frac{D_b}{d_b}|}(t). \tag{9}$$

The pdf $f_{\frac{D_a}{d_a}}(t)$ is given by

$$f_{\frac{D_a}{d_a}}(t) = |d_a| \cdot f_{D_a}(d_a t). \tag{10}$$

The delay deadline d_a belonging to class a traffic is always positive, so that $|d_a| = d_a$. The pdf $f_{\frac{D_a}{d_a}}(t)$ therefore translates to

$$\begin{aligned} f_{\frac{D_a}{d_a}}(t) = d_a &\left(\delta C - (e^\delta - 1) \sum_{j=1}^{h_a(d_a t)} k_a \lambda_a \right) \\ &\cdot \exp\Bigg[-\delta C d_a t + \left((e^\delta - 1) \cdot \sum_{j=1}^{h_a(d_a t)} k_j \lambda_j d_a t \right) \\ &+ \left((e^\delta - 1) \cdot \sum_{j=h_a(d_a t)+1}^{J} k_j \lambda_j (d_a - d_j) \right) \Bigg] \end{aligned} \tag{11}$$

where once again

$$\begin{cases} d_a - d_j \geq t, & j \in [1, h_a(t)] \\ d_a - d_j < t, & j \in [h_a(t) + 1, J]. \end{cases} \tag{12}$$

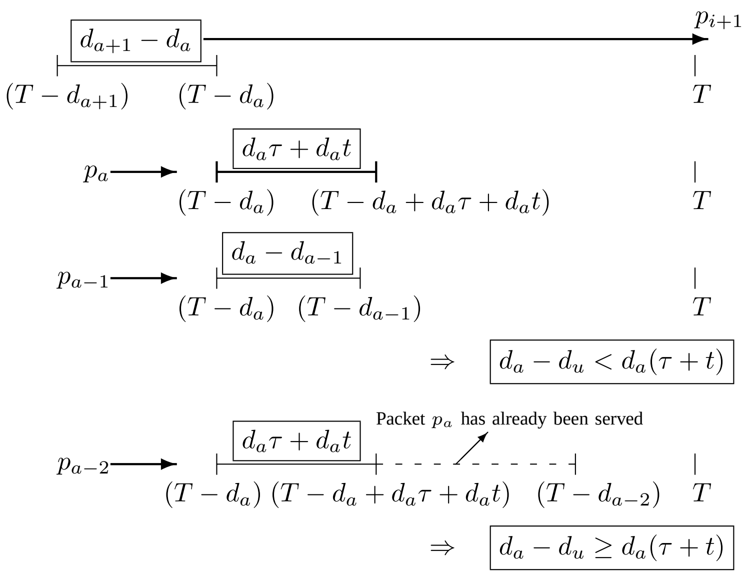

Fig. 1. Timeline for packet p_a.

The next step is to find $f_{\frac{D_a}{d_a}-\frac{D_b}{d_b}}(t)$. Assuming that $\frac{D_a}{d_a}$ and $\frac{D_b}{d_b}$ are independent, this can be obtained through the convolution integral,

$$\begin{aligned} &f_{\frac{D_a}{d_a}-\frac{D_b}{d_b}}(t) = \\ &d_a d_b \cdot \exp(-\delta C d_a t)\cdot\int_{-\infty}^{\infty}\left(\delta C-(e^{\delta}-1)\cdot\sum_{j=1}^{h_a(d_a t+d_a\tau)}k_j\lambda_j\right) \\ &\cdot\left(\delta C-(e^{\delta}-1)\cdot\sum_{j=1}^{h_b(d_b\tau)}k_j\lambda_j\right)\cdot\exp\Bigg[-\delta C(d_a+d_b)\tau \\ &+(e^{\delta}-1)\Bigg\{\sum_{j=1}^{h_a(d_a t+d_a\tau)}k_j\lambda_j d_a(t+\tau)+\sum_{j=h_a(d_a t+d_a\tau)+1}^{J}k_j\lambda_j(d_a-d_j) \\ &\qquad+\sum_{j=1}^{h_b(d_b\tau)}k_j\lambda_j d_b\tau+\sum_{j=h_b(d_b\tau)+1}^{J}k_j\lambda_j(d_b-d_j)\Bigg\}\Bigg]\,d\tau, \end{aligned} \tag{13}$$

Fig. 1 can be consulted to show that the boundary events $h_a(d_a t+d_a\tau)$ and $h_b(d_b\tau)$ in the above equation are correct. These can be rewritten as

$$\begin{cases} d_a-d_j\geq d_a t+d_a\tau, & j\in[1,h_a(n)] \\ d_a-d_j< d_a t+d_a\tau, & j\in[h_a(n)+1,J] \\ & \\ d_b-d_j\geq d_b\tau, & j\in[1,h_b(n)] \\ d_b-d_j< d_b\tau, & j\in[h_b(n)+1,J] \end{cases} \tag{14}$$

Using this information,

$$\begin{aligned} &f_{\frac{D_a}{d_a}-\frac{D_b}{d_b}}(t)=d_a d_b\cdot\exp(-\delta C d_a t) \\ &\cdot\sum_{n=1}^{a+b}\int_{\substack{\tau_{n-1}\\ \tau_0=0}}^{\tau_n}\left(\delta C-(e^{\delta}-1)\sum_{j=1}^{h_a(n)}k_j\lambda_j\right) \\ &\cdot\left(\delta C-(e^{\delta}-1)\sum_{j=1}^{h_b(n)}k_j\lambda_j\right)\cdot\exp\Bigg[-\delta C(d_a+d_b)\tau \\ &+(e^{\delta}-1)\Bigg\{\sum_{j=1}^{h_a(n)}k_j\lambda_j d_a(t+\tau)+\sum_{j=h_a(n)+1}^{J}k_j\lambda_j(d_a-d_j) \\ &+\sum_{j=1}^{h_b(n)}k_j\lambda_j d_b\tau+\sum_{j=h_b(n)+1}^{J}k_j\lambda_j(d_b-d_j)\Bigg\}\Bigg]\,d\tau, \end{aligned} \tag{15}$$

In order to find the fairness pdf of class-a traffic with respect to all other traffic types, we need to find the average:

$$\begin{aligned} &f_{\frac{D_a}{d_a}-\frac{D_b}{d_b}}(t)=\frac{1}{J-1}\sum_{\substack{b=1\\ n\neq a}}^{J}\Bigg\{d_a d_b\cdot\exp(-\delta C d_a t)\cdot\sum_{n=1}^{a+b} \\ &\frac{\left(\delta C-(e^{\delta}-1)\sum_{j=1}^{h_a(n)}k_j\lambda_j\right)\cdot\left(\delta C-(e^{\delta}-1)\sum_{j=1}^{h_b(n)}k_j\lambda_j\right)}{-\delta C(d_a+d_b)+(e^{\delta}-1)\left\{\sum_{j=1}^{h_a(n)}k_j\lambda_j d_a+\sum_{j=1}^{h_b(n)}k_j\lambda_j d_b\right\}} \\ &\cdot\Bigg(\exp\Bigg[-\delta C(d_a+d_b)\tau_n+(e^{\delta}-1)\Bigg\{\sum_{j=1}^{h_a(n)}k_j\lambda_j d_a(t+\tau_n) \\ &+\sum_{j=h_a(n)+1}^{J}k_j\lambda_j(d_a-d_j)+\sum_{j=1}^{h_b(n)}k_j\lambda_j d_b\tau_n+\sum_{j=h_b(n)+1}^{J}k_j\lambda_j(d_b-d_j)\Bigg\}\Bigg] \\ &-\exp\Bigg[-\delta C(d_a+d_b)\underset{\tau_0=0}{\tau_{n-1}}+(e^{\delta}-1) \\ &\quad\cdot\Bigg\{\sum_{j=1}^{h_a(n)}k_j\lambda_j d_a(t+\underset{\tau_0=0}{\tau_{n-1}})+\sum_{j=h_a(n)+1}^{J}k_j\lambda_j(d_a-d_j) \\ &\qquad+\sum_{j=1}^{h_b(n)}k_j\lambda_j d_b\underset{\tau_0=0}{\tau_{n-1}}+\sum_{j=h_b(n)+1}^{J}k_j\lambda_j(d_b-d_j)\Bigg\}\Bigg]\Bigg)\Bigg\}\Bigg|_{t\geq 0} \end{aligned} \tag{16}$$

To find $f_{\left|\frac{D_a}{d_a}-\frac{D_b}{d_b}\right|}(t)$ we map the function from the negative domain into the positive domain, with symmetry around the y-axis. The problem is that the above expression is only defined for $t\geq 0$. In terms of actual measurements performed in a simulation, it only represents values of $\frac{D_a}{d_a}-\frac{D_b}{d_b}$, where $\frac{D_a}{d_a}\geq\frac{D_b}{d_b}$. In order to find values of $\frac{D_a}{d_a}-\frac{D_b}{d_b}$, where $\frac{D_a}{d_a}<\frac{D_b}{d_b}$, we need to look at $f_{\frac{D_b}{d_b}-\frac{D_a}{D_a}}(t)\Big|_{t\geq 0}$ and plot this into the negative domain. The result is

$$f_{\left|\frac{D_a}{d_a}-\frac{D_b}{d_b}\right|}(t) = f_{\frac{D_a}{d_a}-\frac{D_b}{d_b}}(t) + f_{\frac{D_b}{d_b}-\frac{D_a}{d_a}}(t)\Bigg|_{t\geq 0,\text{ for every } b} \tag{17}$$

Equation (18) represents the complete fairness pdf:

$$\begin{aligned}
f_{\left|\frac{D_a}{d_a}-\frac{D_b}{d_b}\right|}(t) = \frac{1}{J-1}\sum_{\substack{b=1\\ n\neq a}}^{J}\Bigg\{ d_a d_b \cdot \sum_{n=1}^{a+b} \\
\frac{\left(\delta C-(e^\delta-1)\sum_{j=1}^{h_a(n)}k_j\lambda_j\right)\cdot\left(\delta C-(e^\delta-1)\sum_{j=1}^{h_b(n)}k_j\lambda_j\right)}{-\delta C(d_a+d_b)+(e^\delta-1)\left\{\sum_{j=1}^{h_a(n)}k_j\lambda_j d_a+\sum_{j=1}^{h_b(n)}k_j\lambda_j d_b\right\}} \\
\cdot\Bigg(\exp\Bigg[-\delta C d_a t-\delta C(d_a+d_b)\tau_n+(e^\delta-1) \\
\cdot\Bigg\{\sum_{j=1}^{h_a(n)}k_j\lambda_j d_a(t+\tau_n)+\sum_{j=h_a(n)+1}^{J}k_j\lambda_j(d_a-d_j) \\
+\sum_{j=1}^{h_b(n)}k_j\lambda_j d_b\tau_n+\sum_{j=h_b(n)+1}^{J}k_j\lambda_j(d_b-d_j)\Bigg\}\Bigg] \\
-\exp\Bigg[-\delta C(d_a+d_b)\underset{\tau_0=0}{\tau_{n-1}}+(e^\delta-1)\Bigg\{\sum_{j=1}^{h_a(n)}k_j\lambda_j d_a(t+\underset{\tau_0=0}{\tau_{n-1}}) \\
+\sum_{j=h_a(n)+1}^{J}k_j\lambda_j(d_a-d_j)+\sum_{j=1}^{h_b(n)}k_j\lambda_j d_b\underset{\tau_0=0}{\tau_{n-1}}+\sum_{j=h_b(n)+1}^{J}k_j\lambda_j(d_b-d_j)\Bigg\}\Bigg] \\
+\exp\Bigg[-\delta C d_b t-\delta C(d_b+d_a)\tau_n+(e^\delta-1) \\
\cdot\Bigg\{\sum_{j=1}^{h_b(n)}k_j\lambda_j d_b(t+\tau_n)+\sum_{j=h_b(n)+1}^{J}k_j\lambda_j(d_b-d_j) \\
+\sum_{j=1}^{h_a(n)}k_j\lambda_j d_b\tau_n+\sum_{j=h_a(n)+1}^{J}k_j\lambda_j(d_a-d_j)\Bigg\}\Bigg] \\
-\exp\Bigg[-\delta C(d_b+d_a)\underset{\tau_0=0}{\tau_{n-1}}+(e^\delta-1) \\
\cdot\Bigg\{\sum_{j=1}^{h_b(n)}k_j\lambda_j d_b(t+\underset{\tau_0=0}{\tau_{n-1}})+\sum_{j=h_b(n)+1}^{J}k_j\lambda_j(d_b-d_j) \\
+\sum_{j=1}^{h_a(n)}k_j\lambda_j d_a\underset{\tau_0=0}{\tau_{n-1}}+\sum_{j=h_a(n)+1}^{J}k_j\lambda_j(d_a-d_j)\Bigg\}\Bigg]\Bigg)\Bigg\}\Bigg|_{t\geq 0}.
\end{aligned} \tag{18}$$

To find an expression for the stochastic fairness bound of EDF traffic, we could now substitute (18) into (6), which for the sake of convenience has been represented here

$$\int_{F_a(\delta)}^{\infty} f_{\left|\frac{D_a}{d_a}-\frac{D_b}{d_b}\right|}(t)\,dt = \delta. \tag{19}$$

The result is unfortunately a non-linear equation, which is difficult to solve analytically for varying values of δ. Instead we solve it numerically.

V. Results

A. Simulation Model

In order to verify the precision of the derived analytical expressions, we created a custom-made simulation package using Borland C++. The analytical expressions were evaluated in MATLAB. The parameters used for the simulation model and analytical expressions were conveniently kept the same as in [1] and [3], as this work is based on these papers. The parameters are summarised in Table I.

TABLE I
System parameters

Type of traffic	d_i (ms)	k_i	ρ_i (Mbps)
T0 — Audio	6	200	0.064
T1 — Video conference	10	82	0.5
T2 — Stored video	14	15	3

where

d_i = delay deadline
k_i = number of independent sources
ρ_i = mean rate of Poisson sources
C = link capacity = 100Mbps
L = packet size = 10Kb
Buffer Capacity = ∞
Serving Interval = $10\mu s$

The simulation model used resulted in a load of 98.8%. The simulation was run for 24 hours of network time. As the serving interval is $10\mu s$, this corresponds to 8.64×10^9 scheduling intervals. During this time interval 100,000 samples were taken at regular intervals to produce the fairness distribution plots.

B. Comparison of Analytical and Simulation Results

In Fig. 2, the first 3 diagrams contain the pdf distributions of $f_{\frac{D_a}{d_a}-\frac{D_b}{d_b}}(t)$ for the 3 traffic classes, the second 3 diagrams contain the pdf distributions of $f_{\left|\frac{D_a}{d_a}-\frac{D_b}{d_b}\right|}(t)$, while the last 3 diagrams contain the actual stochastic fairness bound distributions for varying values of δ, the maximum probability with which the fairness bound F_a may be exceeded.

If we compare the fairness of the various traffic classes in Figs 2(g), 2(h), and 2(i) we find that as we approach the deterministic case of $\delta = 0$, the fairness bound increases dramatically. As Figs. 2(a)–2(c) indicate, the fairness pdf actually falls off exponentially. A deterministic fairness bound therefore does not exist.

Referring once again to Figs. 2(g)–2(i), for 99.9% of the time, at a value of $\delta = 0.001$, both the simulation and analytical curves indicate that the traffic classes 0, 1 and 2 respectively experience fairness bounds of approximately

4.1, 3.8, and 3.6. In this case traffic class 2 experiences slightly better fairness treatment, while traffic class 0 is treated the least fair. On the other extreme, for 95% of the time, represented by $\delta = 0.05$, the fairness curves of classes 0–2 lie at values around 1.3, 1.2, and 1.1, respectively. Once again, EDF treats class 2 the fairest, followed by class 1 and finally class 0.

VI. Conclusion

In this paper we proposed a new stochastic fairness definition suitable for real-time traffic. We used the queueing delay probability density function derived in [1] to find the fairness behaviour of real-time traffic classes in an EDF queueing system. The result is a model that is able to adequately approximate the stochastic fairness behaviour of an EDF queueing system. This model has showed us that all traffic classes are treated in a similar fashion, although traffic class 2 experiences slightly better and class 0 slightly worse treatment than class 1. Furthermore, we noted that the fairness of EDF can not be bounded deterministically.

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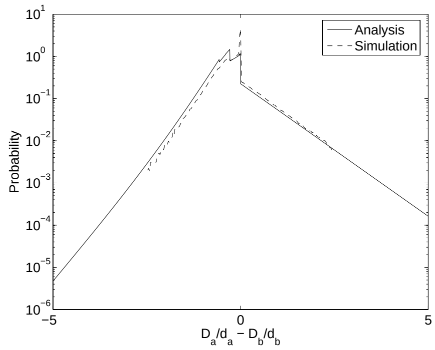
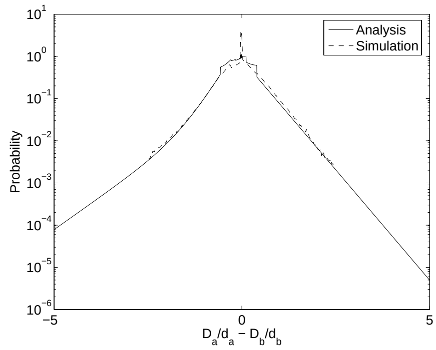
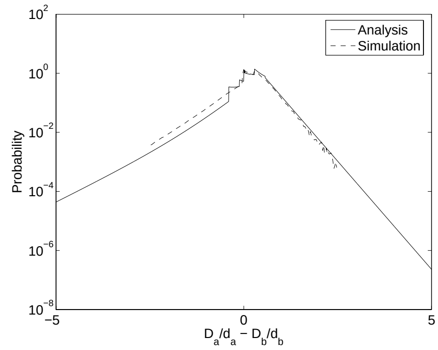


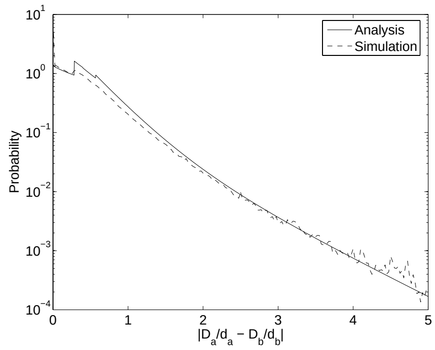
(a) $f_{\frac{D_a}{d_a}-\frac{D_b}{d_b}}(t)$ for Type 0 traffic



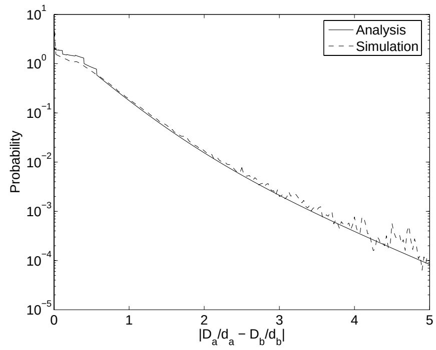
(b) $f_{\frac{D_a}{d_a}-\frac{D_b}{d_b}}(t)$ for Type 1 traffic



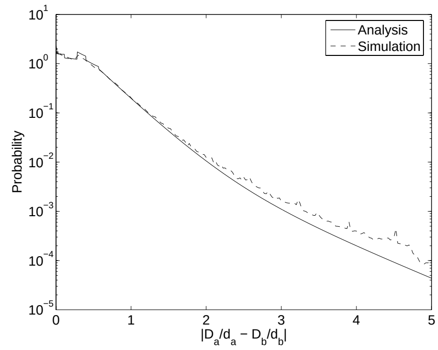
(c) $f_{\frac{D_a}{d_a}-\frac{D_b}{d_b}}(t)$ for Type 2 traffic



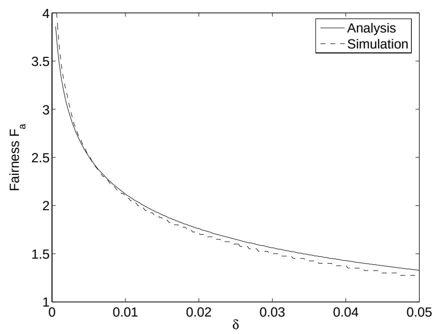
(d) $f_{|\frac{D_a}{d_a}-\frac{D_b}{d_b}|}(t)$ for Type 0 traffic



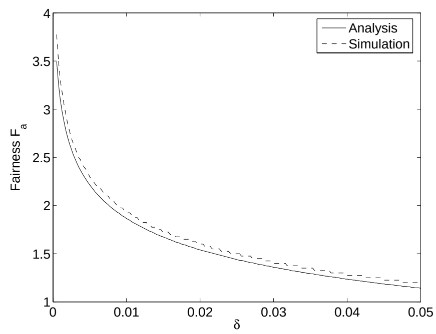
(e) $f_{|\frac{D_a}{d_a}-\frac{D_b}{d_b}|}(t)$ for Type 1 traffic



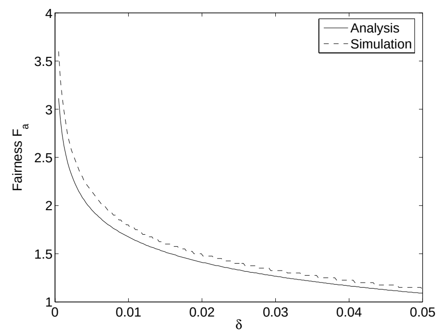
(f) $f_{|\frac{D_a}{d_a}-\frac{D_b}{d_b}|}(t)$ for Type 2 traffic



(g) Type 0 statistical fairness distribution



(h) Type 1 statistical fairness distribution



(i) Type 2 statistical fairness distribution

Fig. 2. Normalised fairness curves