Multiband Support Recovery in 1.2GHz Bandwidth using the Modulated Wideband Converter

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Abstract- This paper deals with a sub-Nyquist sampling method, the Modulated Wideband Converter (MWC) that can efficiently recover the support for sparse wideband analog signals. This approach can be used in spectrum sensing and blind sub-Nyquist support recovery of the frequency bands for cognitive radio (CR) operation. The simulation results obtained for MWC of 1.2GHz bandwidth using ModelSim which is a hardware description language is presented.

Index Terms— Spectrum Sensing, Support Recovery, Cognitive Radio

I. Introduction

A wide range of frequencies in a spectrum is collectively described by a relative term in communication called ”wideband” while the task of identifying the frequency support of a specific input signal is addressed by the term “spectrum sensing”. This essentially includes those spectrum intervals where the noise floor is exceeded by the signal power. Shannon sampling theorem [1] is used to obtain the Nyquist rate for input sampling as assumed by traditional spectrum sensing methods. Even though this theorem suggests a rate equal to twice the maximum frequency present in the signal, it could be noted that a practically viable rate may be even higher than this. If it is possible for a system to identify the unknown spectral support without the knowledge of carrier frequency or the sampling method, then that type of system can be called as a spectrum blind system.

Cognitive Radio receivers [2] use spectrum sensing in opportunity based utilization of unused frequency regions. In a commercial point of view, a spectrum sensing mechanism that can react on a real time basis to the cognitive decisions is greatly demanded by the CR technology. [3] and [4] refers to a range of spectrum sensing methods that primarily require the presence of transmitted signals. On the other hand, analog to digital conversion (ADC) [5] technologies that support bandwidths in the range of gigahertz make Cognitive Rradio communications expensive. Compressive Sampling (CS) is applied to reduce the bandwidth requirements thereby making it less expensive. The recovery process turns out to be non-deterministic in nature.

In this paper we mainly focus on the identification of frequency support in sparse analog signals using sub-Nyquist sampling for use in CR. In section 2, we give a background on Compressive Sensing (CS) and CR. In section 3 we will introduce the MWC system. In section 4, the simulation results obtained for 1.2GHz frequency band is depicted along with the parameters chosen. We conclude with a discussion on the data we collected in section 5 and summarize the rate at which the Nyquist rate is reduced using MWC in section 6.

II. Background

A. Compressive Sampling

Compressive Sampling, also known as Compressed Sensing refers to the technique of recovering signals by much fewer samples than that suggested by Nyquist rate [6]. There are two major principles on which CS is based; one is sparsity pertaining to the Signals of Interest (SOI) and the other one is incoherence pertaining to the sensing modality [7].

Sparsity determines the efficiency by which signals can be acquired non-adaptively as it expresses an idea regarding required information rate of a continuous time signal and that may be much less than that suggested by its bandwidth. Many signals are compressive in the sense that they can be concisely represented when expressed in the proper basis Ψ [5] and this fact is exploited by the CS technique to a large extend.

Incoherence extends the duality between time and frequency and the idea that objects having a sparse representation in Ψ must be spread out in the domain in which they are acquired. A major concern of CS is low coherence pairs. Extremely dense representations of sampling waveforms in Ψ makes incoherence as important as sparsity.

Comparative measurements with basis Φ used by CS is smaller and different from that with basis Ψ representing mostly sparse signals and contains all the useful

information [7]. Therefore recovery of the SOI involves solving an indeterminate system of equations since the number of measurements taken is smaller than the dimensionality (order) of the original signal. This indeterminate system of linear equations can be solved if the original signal is limited to be sparse, and the sparsest solution of the CS problem given in [8] is as follows

$$\min_{v \in R^m} \|v\|_0 \text{ subject to } \Phi v = y \qquad (1)$$

where $||v||_0$ is the size of the support of v, Φ is the sampling matrix with rows $\varphi_j \in R^m$ and y is the sampled vector. However, this minimization problem is NP- hard (non-deterministic polynomial-time hard). This problem of sparse representation can be solved using *l1*-norm minimization by converting it into a linear problem. The problem of *l1*-norm minimization as given in [6, 7] is

$$\min_{v \in R^m} \|v\|_1 \text{ subject to } \Phi v = y \qquad (2)$$

where $||v||_1 = \sum_j |v_j|$. Technically it is proved that for many problems *l1*-norm minimization is equivalent and easier than (1) and also that solving using the *l1*-norm minimization leads to unique solution of (1).

B. Cognitive Radio and Spectrum Sensing

Traditional Communication systems such as television, radio stations, mobile carriers, air traffic control etc, are allotted with predetermined frequency bands and over the years the government has allocated the majority of the available spectrum to legacy users, reserving a particular frequency interval for each owner. This resource allocation strategy has led to spectrum congestion and the demand for permanent transmission bands can only be rarely satisfied. A recent study on the utilisation of spectrum by the Federal Communications Commission (FCC) in the United States and by similar agencies in other countries indicates that most of the spectrum remains underutilized in a given geographical location and time duration corresponding to a very small number of concurrent transmission by legacy users.

As per the definition of FCC, Cognitive Radio refers to a system that senses its operational electromagnetic environment and can dynamically and autonomously adjust its radio operating parameters to modify system operation, such as maximize throughput, mitigate interference, facilitate interoperability, access secondary markets etc. Spectrum sensing therefore takes place whenever the CR searches for available transmission holes. After a certain frequency band is chosen, the CR continuously monitors the spectrum in order to detect any change in the activity of the primary users. Once a primary user becomes active, the CR must choose another working band, or tailor its transmission to reduce in-band power. Quick and efficient spectrum sensing is evidently an essential component of CR functionality.

Recently, there has been growing interest in spectrum sensing for mobile cognitive radio (CR) receivers, which aim at utilizing unused frequency regions on an opportunistic basis. Commercialization of CR technology necessitates a spectrum sensing mechanism that reacts in real time to cognitive decisions. Sensing in CR mobiles must be performed using minimal hardware and software resources. Therefore, enabling widespread use of CRs calls for innovative spectrum sensing techniques.

C. Wideband Spectrum Sensing by Cognitive Radio

In communications, wideband is a relative term used to describe a wide range of frequencies in a spectrum. Spectrum sensing refers to the task of identifying the frequency support of a given input signal. The frequency support then consists of those spectrum intervals in which the signal power exceeds the noise floor. Also frequency support can be defined as the indices of active bands in a spectrum. In a fully spectrum blind system, the digital algorithms developed for processing the samples identifies the unknown spectral support.

III. MODULATED WIDEBAND CONVERTER

The Mixed Analog Digital system described below operates at sub-Nyquist rate and has much lower complexity than either a traditional sampling system or CS recovery techniques. MWC differ from most standard approaches to the problem of CS recovery as it does not assume that the signals of interest (SOI) are confined to a discrete set of frequencies and do not rely on traditional recovery methods such as *l1* minimization.

Modulated Wideband Converter (MWC) presented in [9, 10, 11] is a mixed analog digital system which has an analog front end and a digital back end. The main function of the analog front end is to alias the spectrum, such that a spectrum portion from each band appears in base band. MWC uses spread spectrum techniques. The advantages of this system with respect to current architectures are threefold. First, the analog front-end is fixed. Second, both the ADC and the DSP rates are substantially below Nyquist. Finally, the sensing resources are shared with the reception path of the CR, so that the low rate streaming samples can be used for communication purposes of the device, besides the sensing functionality they provide. Combining these advantages leads to a real time map of the spectrum.

Our research is based on the MWC system, which enables baseband processing using a set of random demodulators, generating a low rate sequence corresponding to any information band of interest from multiband analog signals, without going through high Nyquist rate. These low rate digital samples enter CTF block which uses a digital algorithm for support recovery.

A. *Sampling Stage*

In practical cases the Fourier transform of wideband signals occupies only a small portion of a wide spectrum, with unknown frequency support. In wideband communication, the receiver sees the sum of several radio-frequency transmissions. Each signal is modulated around an unknown and different carrier frequency.

Formally, the Fourier transform of *x(t)* is given as

$$X(f) = \int_{-\infty}^{\infty} x(t)e^{-j2\pi ft}\,dt \qquad (3)$$

is zero for every f is not an element of F where $F = [-1/2T_{nyq}, +1/2T_{nyq}]$.

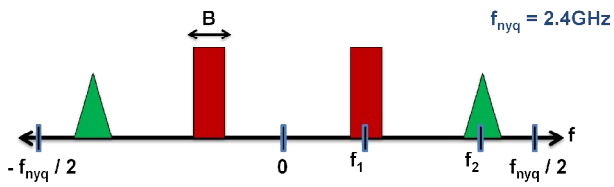

Fig 1: Spectrum of a multiband signal

The sampling stage [10, 11] consists of an RF front end which includes a bank of multiple channels in which the input signal enters simultaneously. Each channel consists of a modulator and a low pass filter. Following the RF front end is a low rate ADC on each channel which handles low analog bandwidth.

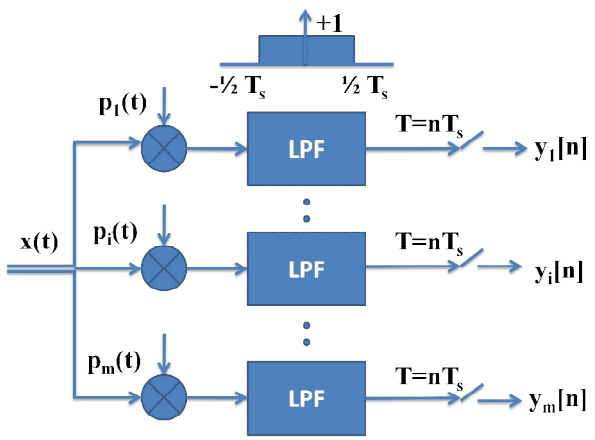

Fig 2: Modulated Wideband Converter

In each channel, the signal gets multiplied by a mixing function, $p_i(t)$ which is a periodic random bit sequence taking ± 1 values that share a common period T_p seconds. The period T_p is assumed as some integer multiple of the Nyquist period $T_p = L/f_{nyq}$.

$$p_i(t) = \alpha_{ik,} \qquad (4)$$

$$where\ k\frac{T_p}{M} \le t \le (k+1)\frac{T_p}{M}\ and$$

$$0 \le k \le M-1$$

with $\alpha_{ik} \in \{+1, -1\}$ and $p_i(t + n\,T_p) = p_i(t)$ for every $n \in \mathbb{Z}$.

The Fourier expansion of p_i (t) is given as

$$P_i(f) = \int_{t=-\infty}^{\infty} p_i(t)e^{-j2\pi ft}dt$$

$$= \Sigma_{l=-\infty\ to\ +\infty}\, c_{il}\,\delta\,(f - l/T_p) \qquad (5)$$

where δ is the Dirac delta function and

$$c_{il} = \frac{1}{T_p}\int_0^{T_p} p_i(t)e^{-j\frac{2\pi}{T_p}lt}dt$$

The purpose of this mixing is to spread the spectrum. The modulated signal is then filtered by a low pass filter with cut off $f_s/2$. Then the filtered signal is sampled uniformly at every $T_s = 1/f_s$ second. As the system consists of m channels the average sampling rate is m/T_s samples per second.

The analog multiplication of $x(t)$ with $p_i(t)$, $\tilde{x}_i(t)=x(t)p_i(t)$ will be transformed to convolution while expressed in frequency domain.

$$\widetilde{X}_i(f) = X(f)P_i(f)$$

$$= \sum_{l=-\infty}^{\infty} c_{il}X(f - lf_p) \qquad (6)$$

From (6) it is clear that $\tilde{X}_i(f)$ is a linear combination of f_p shifted copies of $X(f)$. The modulated output is then low pass filtered with $H(f)$ having an ideal rectangular function as its frequency response. The interval for the *rect* function is $[-1/2T_s, +1/2T_s]$.

The DTFT of the ith sequence $y_i[n]$ is given as

$$Y_i(e^{j2\pi fT_s}) = \sum_{l=-L_0}^{+L_0} c_{il}\,X(f - lf_p), \qquad f \in \mathcal{F}_s \qquad (7)$$

where L_0 is the smallest integer satisfying $2L_0 + 1 \ge f_{nyq}/f_p$.

The relation between unknown $X(f)$ with that of $y_i[n]$ can be seen from (7). Therefore

$$y_i(f) = Y_i(e^{j2\pi f T_s}) \; ; 1 \leq i \leq m. \tag{8}$$

The output obtained from i^{th} channel is represented as $y_i[n]$. Commercial ADCs can be used for sampling purpose due to the low sampling rate appearing in each channel.

B. Recovery Stage

Support recovery in MWC is done with the help of a CTF block. The input to the CTF block is the low rate digital samples from the sampling stage.

This paper aims to recover the support of active bands from the input data $y(f)$. The support recovery algorithm is referred as the Continuous-To-Finite (CTF) [10] block given in Fig 3.

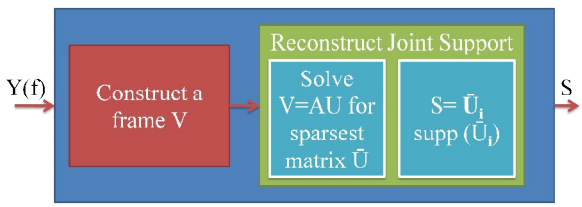

Fig 3: Continuous To Finite Block

The CTF constructs a frame from the input samples i.e. a set of y values. Such a frame can be obtained by computing [12]

$$Q = \sum_{n=-\infty}^{+\infty} y[n]\, y^T[n] \tag{9}$$

Then it solves a finite dimensional sparse representation problem, from which it identifies spectrum slices containing signal energy. Thus the digital recovery algorithm could be used for executing and detecting the correct support set and input carrier positions. The CTF block has a major role in signal reconstruction beyond the robustness in estimating spectral support. Once spectral support is known, real time processing and reconstruction is possible, that is at the (low) speed of the streaming measurements.

IV. Our Approach

Mixed analog-digital spectrum sensing method uses MWC for CS and CTF block for support recovery that is especially suited for the typical wideband setting of CRs. We have implemented a mathematical model of Mixed-analog-digital system using Modelsim. Modelsim is a very powerful and versatile HDL (Hardware Description Language) simulation tool. It helps in integrating to FPGA by creating a VHDL test bench.

The flow diagram given in Fig 4 shows that the output from the signal representation block undergoes sub-Nyquist sampling using MWC and then the support recovery via CTF block. Some of the random functions like mixing function, original support are calculated using MATLAB. Then the values are written into a text file and by using ModelSim, these values are read.

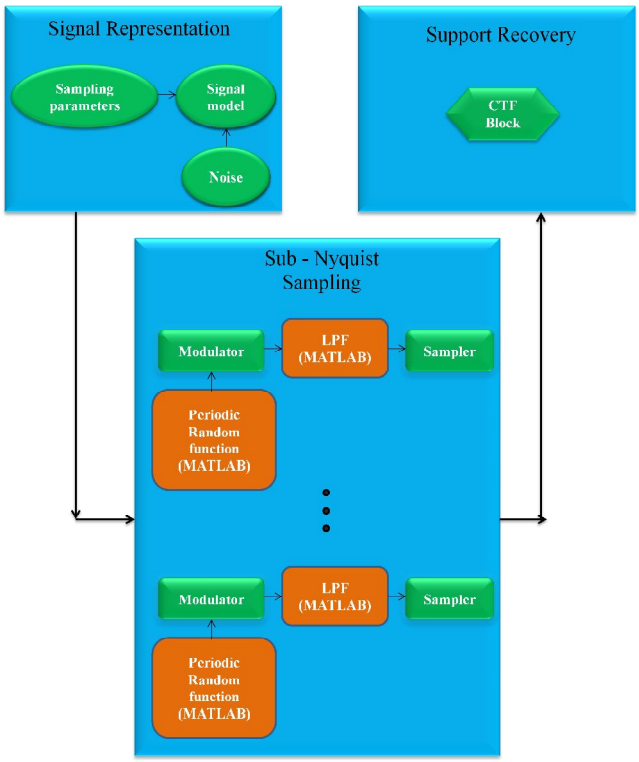

Fig 4: Flow Diagram

Our main aim is support recovery as described in section 3. Parameter choices of our MWC for 1.2GHz are given in Table 1. Also the input SNR to the input signal is considered as 10. The number of bands and the bandwidth for multiband signal under consideration is selected as 4 and 1.2GHz respectively. Therefore the Nyquist frequency will be 2.4GHz. For perfect blind reconstruction [9, 10] of a multiband signal, the bandwidth, B for each of the band in multiband must satisfy $2NB < f_{nyq}$ and the number of channels, m must be $m \geq 2N$. To avoid edge effects $f_s \geq f_p \geq B$ must be ensured.

From (7), L_0 is chosen as the smallest integer such that the sum contains all nonzero bands in the multiband signal model. L_0 can be calculated by

$$L_0 = \left[\frac{f_{nyq} + f_s}{2f_p}\right] - 1 \qquad (10)$$

Decimation factor, L is therefore given as $L = 2L_0 + 1$. The number of flops in the sign alternating function, M can be calculated from [10]

$$M \geq 2\left[\frac{f_{nyq}}{2f_p} + \frac{1}{2}\right] - 1 \qquad (11)$$

Table 1. Parameter Choices for 2.4GHz band MWC

Parameter Notation	Description	Choice
N	Number of bands	4
B	Maximum width of each band	25MHz
f_{nyq}	Nyquist frequency	**2.4GHz**
L	Decimation factor	51
f_p	Frequency of $p_i(t)$	f_{nyq} / L
m	Number of sampling channels	10
f_s	Sampling rate	f_p
M	Number of ±1 intervals in each period of $p_i(t)$	L

The input signal along with added noise is mixed with random periodical functions. The mixing helps in aliasing the energies to baseband and thereby the baseband operations are enabled without losing information. This mixed output from each channel is then filtered using an ideal LPF having cutoff, $1/2T_s$ = 23.5MHz. Then the high rate input sequences (Length of input signal = 1836 samples for each channel) are then down sampled to low rate ones (Length of input signal/ L = 36 samples for each channel).

The CTF block helps in support recovery by frame construction followed by replacing the infinite sum by a finite sum.

The original support is the indices of active bands. The recovered support is the result we obtained from CTF block. From Table 2, we can see that the original support set consists of 4, 5,19,33,47 and 48 where as recovered support set consists of 1, 4, 5,19,33,47, 48 and 51 as the active supports. We consider the result as success if original support set is the subset of recovered support set. Here as we can see all the original supports are identified, it is a successful recovery.

Table 2. Support Recovery using MWC

Original Support Set	Recovered Support Set
{ 4, 5, 19, 33, 47, 48 }	{ 1, **4**, **5**, **19**, **33**, **47**, **48**, 51 }

The probability for the occurrence of false positives for a successful recovery is given in the pie diagram shown in Figure 5. A false positive is that support which is present in recovered support set but not in original support. 15% of the simulation results show that the original support is equal to the recovered support, 50% shows that the recovered support set contains 2 false positives along with the original support set and 35% shows 4 false positives in recovered support set.

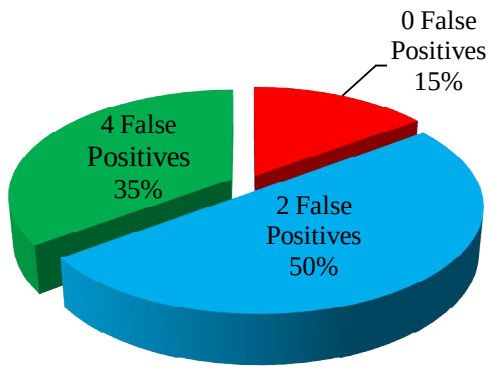

Fig 5: Probability for the occurrence of False Positives

The probability for the occurrence of false negatives for a failed recovery is given in the pie diagram shown in Figure 6. A false negative is the support which is present in original support set but not in recovered support set. 63% of the simulation results show that 2 of the original support were missing when compared with the recovered support, 32% shows that the original support set contains 4 false negatives and 5% shows 6 false negatives in original support set.

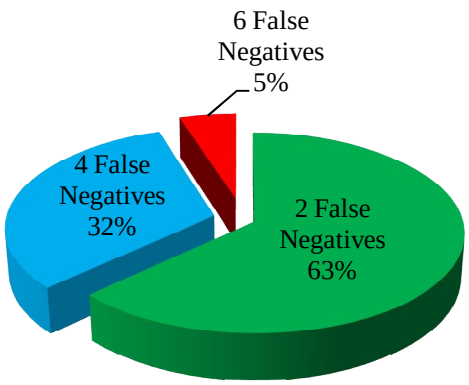

Fig 6: Probability for the occurrence of False Negatives

V. Conclusion on Data

We repeated the simulations for more than 2500 times for the above chosen parameters. The results showed successful support recovery for about 80% of the simulations and 20% showed failed recovery. As 80% showed success, the MWC is considered to be an apt technique for the support recovery of a multiband signal. We conclude that this 80% success can be raised by selecting more appropriate parameters.

VI. Summary

In this paper we showed the ModelSim simulation results for MWC, demonstrating its ability to sample and recover support at significantly below Nyquist rate. MWC is a multi-channel system which samples sparse multiband signals. Here we used 10 low-rate sampling channels to sample the 1.2GHz multiband signal. The Nyquist rate for the multiband signal is $f_{nyq} = 2 * 1.2GHz = 2.4GHz$. The sampling rate for each channel is $fs = 2.4GHz/51 = 47.05MHz$. Therefore, the actual sampling rate is 47.05MHz*10 = 470.5MHz. This shows that the sampling rate is reduced to 80% than Nyquist rate by using MWC.

VII. Acknowledgments

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